Shormann Algebra 1
with Integrated Geometry

Connecting Math to Your World
and Your Creator

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ISBN: 978-0-989010-4-6

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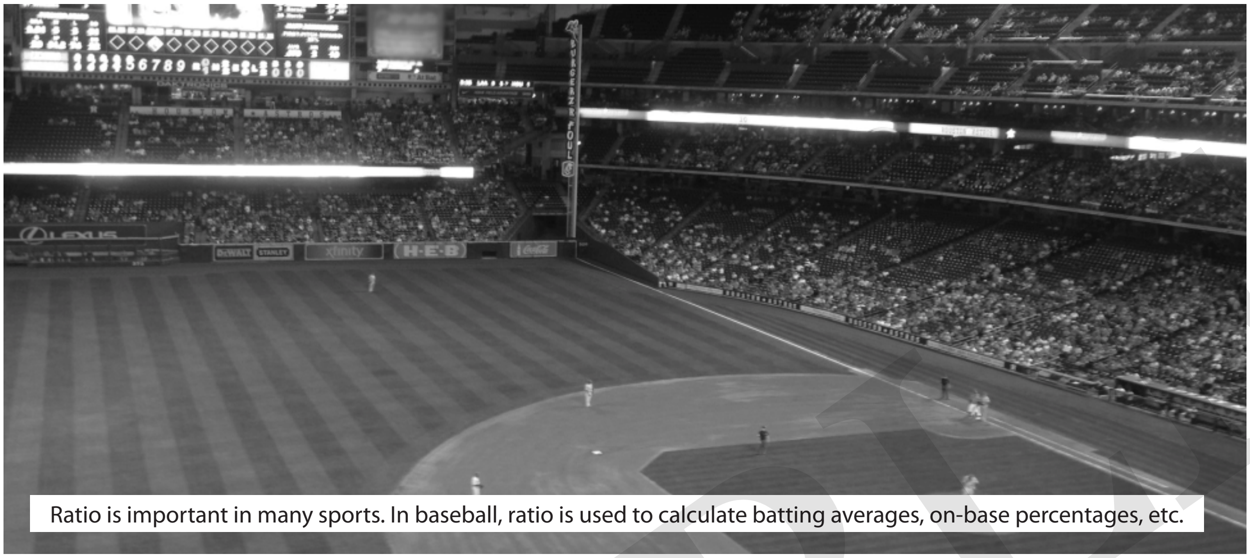


Ratio is important in many sports. In baseball, ratio is used to calculate batting averages, on-base percentages, etc.

Rules and Definitions

Rules

Table 5.1

Fraction/Decimal/Percent conversions (memorize these):

Fraction	Decimal (0.xx)	Percent (xx%)
1/2	0.50	50%
1/4	0.25	25%
1/8	0.125	12.5%
1/3	$0.\overline{3}$	$33.\overline{3}\%$
1/6	$0.1\overline{6}$	$16.\overline{6}\%$
1/5	0.2	20%
1/10	0.1	10%

The *reciprocal* of x is $\frac{1}{x}$

Definitions

- %: symbol for percent, which is an abbreviation for "per 100". For example 50% is the same thing as writing $\frac{50}{100}$, it's just faster to write or type '50%'.

5A Fraction/Decimal/Percent

One of the most confusing things about learning fractions is that they come in so many different forms! For example, $\frac{3}{10}$ is the same thing as 30%, 0.3, 3/10 or the ratio 3:10. On top of that, in baseball, if you get on base an average of 4 out of every 10 times at bat, you are doing pretty good, but then baseball statisticians don't say your on-base percentage is 40%, they say it is 0.400! They call it "percentage", but then write it in decimal form.

Confused? Don't be! All these different formats are really just different ways of expressing ratios, and as we discussed in Lesson 4 and in the definitions above, the different forms allow us to simplify how we write fractions. Remember, math is a language (the language of science), and with languages, we have different ways of saying the same thing. For example, you may start the day with a list of chores to complete, and your mother says "Please finish all your chores before you play." Now, I'm sure you would never do this, but there are other children who whine and complain about their chores, and end up not doing them, which exasperates mom until she says "Do your chores, NOW!!!" See? Different ways of saying the same thing! Don't think of fraction/decimal/percent as anything more than 3 ways to say the same thing. Table 5.1 above contains some commonly used fraction/decimal/percent conversions. Your goal is to memorize the table. Math is about applying rules, so the more rules you memorize, the faster and more accurate you will be with your math.

Example 5.1 Fill in the empty cells of the following table: Round missing decimal and percent values to 2 decimal places. You may use a calculator.

Fraction	**Decimal**	**Percent**
	0.75	
		62.5%
1/7		

solution: **Row 1**: Use Table 5.1, or remember that 1/4 = 0.25 = 25%. From this, it follows that 3/4 = 0.75 = 75%.
Row 2: As with Row 1, look for patterns related to values in Table 5.1. Notice that 12.5 x 5 = 62.5. Therefore, since 1/8 = 0.125 = 12.5%, then 5/8 = 0.625 = 62.5%
Row 3: This value is not in Table 5.1. You may use a calculator, because the goal of the problem is to see the relationship between fraction/decimal/percent, and not whether you remember how to do long division by hand. You should be able to, but that's not the point here. Performing 1÷7 on a calculator, you should get 0.142857, which you can round to 0.14 for the decimal form, and 14.29% for the percent.

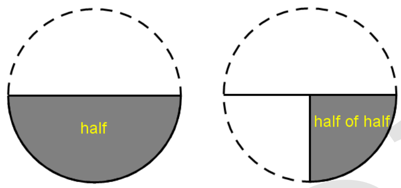
5B Operations With Fractions and Decimals (Multiplication and Division)

We covered addition and subtraction of fractions and decimals in Lesson 4C, and multiplication and division of decimals in Lesson 2. Here we will consider how multiplication and division of fractions works.

Multiplication of fractions: To multiply fractions, we multiply the numerators together, AND we multiply the denominators together. This is different from addition and subtraction, where, as long as the denominators are equal, the answer has the same denominator as the original fractions. For example, consider the following:

$$\frac{1}{2}\cdot\frac{1}{2}=\frac{1\cdot 1}{2\cdot 2}=\frac{1}{4}$$

When multiplying fractions together, instead of thinking "half times half", think "half of half", which encourages us to think of taking half of something, and splitting that in half, like this:



Example 5.2 Multiply. Reduce if necessary. a) $\frac{1}{4}\cdot\frac{1}{3}$ b) $\frac{3}{5}\cdot\frac{5}{8}$ c) $\frac{7}{8}\cdot\frac{4}{7}$

solution: a) $\frac{1}{4}\bullet\frac{1}{3}=\frac{1}{12}$

b) First, get $\frac{3\cdot 5}{5\cdot 8}$. Next, remember that order doesn't matter in multiplication (Lesson 3), and that any number divided by itself equals 1 (Lesson 4). Therefore, 5/5 equals 1, so we can just cancel the 5's. This is easier than multiplying to get 15/40, because we will have to reduce by dividing by the GCF of 5. Why multiply by 5 and then divide by 5? Save time by canceling!

$$\frac{3\cdot \cancel{5}}{\cancel{5}\cdot 8}=\frac{3}{8}$$

c) Multiply, cancel the 7's, and reduce by dividing by the GCF of 4.

$$\frac{\cancel{7}\cdot 4}{8\cdot \cancel{7}}=\frac{4}{8}=\frac{1}{2}$$

The word *of* is helpful when multiplying fractions. In fact, when given any word problem involving fraction multiplication, we can set it up using the memory aid (mnemonic)

$$F\cdot a=b$$

Pronounced "*F* of *a* equals *b*", where *F* stands for *fraction*. We could also substitute in "D" for decimal, or "P/100" for percentage. You can usually tell which word problems will require the use of F · a = b, because they always give you all but one of the three parts.

Example 5.3 1/4 of 60 equals what number?

solution: Using F · a = b, think "F of a equals b, so 1/4 of 60 equals b". Now you know you need to multiply 1/4 by 60, which is the same thing as dividing 60÷4. I will assume you know how to divide 4 into 60 to get 15. **1/4 of 60 equals 15**.

Example 5.4 0.2 of 35 equals what number?

solution: Using D · a = b, think "D of a equals b, so 0.2 of 35 equals b". Ignoring the decimals until the end (Example 1.11), multiply 2 by 35 to get 70. Add the decimal back in to get **0.2 of 35 = 7**.

Example 5.5 20% of 35 equals what number?

solution: Using P/100 · a = b, think "P/100 of a equals b, so 20/100 of 35 equals b". You can save a lot of time on this problem by converting 20% to decimal form (0.2). This makes multiplication easier. Also, it makes the problem identical to Example 5.4! Therefore, the answer is **7**.

__Division of fractions__: The best way to divide fractions is to multiply the dividend by the reciprocal of the divisor. For example, we can solve 2 ÷ 1/2 as follows:

$$2 \div \frac{1}{2} = \frac{2}{1} \div \frac{1}{2} = \frac{2}{1} \times \frac{2}{1} = \frac{4}{1} = 4$$

A simple way to think about dividing by fractions is **"invert and multiply."** Also, think about how 2 ÷1/2 = 4. When dividing, think "how many 1/2 pieces are in two whole pieces?" Or maybe think "how many half-pies are in two whole pies?" Well, there are 4 half pies in two whole pies!

Example 5.6 Divide. a) $4 \div \frac{1}{2}$ b) $\frac{1}{3} \div \frac{1}{8}$ c) $\frac{3}{5} \div \frac{3}{10}$

solution: a) Write whole numbers like 4 as fractions, then invert and multiply. Also, think "how many 1/2 pieces are in 4 whole pieces?

$$\frac{4}{1} \cdot \frac{2}{1} = \frac{8}{1} = 8$$

b) $\frac{1}{3} \cdot \frac{8}{1} = \frac{8}{3}$

c) When you invert and multiply, notice that the 3's cancel:

$$\frac{\cancel{3}}{5}\cdot\frac{10}{\cancel{3}}=\frac{10}{5}=2$$

In Lesson 2, we reviewed long division with a remainder. We also practiced division with decimal quotients, but let's work on that a little bit more.

Example 5.7 Solve. Round answers to 2 decimal places. a) 169 ÷ 4   b) 65÷6

solution: Instead of writing the dividends as whole numbers, be prepared to add decimal place holders. For example, since you are writing answers to two decimal places, instead of "169", write "169.00." Sometimes, you may need to add more and then round back to the required dec. places.

```
a)    42.25            b)   10.8333     which rounds to 10.83
   4)169.00               6)65.0000
     16                     6
     009                    05
      - 8                   - 0
        10                    50
       - 8                   -48
         20                    20
       - 20                   - 18
          0                      20
                                -18
                                  20
                                 -18
```

Practice Set 5 (subscripts tell you which lesson each problem came from)

You may use a calculator for problems 1 & 2 only.

1_5. Fill in the empty cells of the following table: Round missing decimal and percent values to 1 decimal place. You may use a calculator.

Fraction	Decimal	Percent
1/5		
	0.35	
		40%

2_5. Fill in the empty cells of the following table: Round missing decimal and percent values to 2 decimal places. You may use a calculator.

Fraction	Decimal	Percent
1/4		
	$0.1\overline{6}$	
		23%

3_5. Multiply. a) $\frac{1}{6} \cdot \frac{1}{9}$ b) $\frac{3}{9} \cdot \frac{9}{8}$

4_5. What fraction of 60 equals 30? Hint, if $F \cdot a = b$, then $F = b \div a$.

5_5. What fraction of 60 equals 40?

6_5. 30% of 80 is what number?

7_5. 0.25 of 80 is what number?

8_5. Divide. a) $10 \div \frac{1}{9}$ b) $\frac{2}{7} \div \frac{2}{8}$

9_5. Divide and round to 2 decimal places. $54 \div 40$

10_5. Divide and round to 2 decimal places. $255 \div 31$

11_4. Find the GCF between a) 8 and 18 b) 40 and 11

12_4. Reduce the following. a) $\frac{8}{18}$ b) $\frac{11}{40}$

13_4. Find the LCM of 3, 4, and 10.

14_4. Add. $\frac{1}{4} + \frac{1}{3} + \frac{7}{10}$

15_4. Add. 42 + 18.013

16_3. Simplify. $-(-(-(-4)))$

17_3. Add algebraically. $2 - 4 + 7 - 11 - (-14)$

18_3. Simplify. $-2^2 + 3^2(8 + |-6|)$

19_2. Convert 84 to Roman numerals.

$20_{3,2}$. Simplify. $8i^2 - |-3| + 2(4-1)^2$

Lesson 10 Geometry, Part II

A compass and a straightedge, the two tools of the Greek geometer.

Rules and Definitions

Rules

No new rules for Lesson 10.

Definitions

- mathematical proof: A deductive argument where rules and definitions are applied to reach a logical conclusion.

- theorem: Also called *propositions*, these are true mathematical statements requiring proof. Theorems particular to a certain discipline are sometimes called *lemma*, such as the lemma found in Newton's landmark science book, *Principia*.

Materials required for Lesson 10: Compass, straightedge (ruler), Geometer's Sketchpad software (optional).

10A Inductive Reasoning, Construction

Inductive reasoning(See definition in Lesson 1): Some of God's first commands to humans included being fruitful and multiplying, and ruling over Creation (Genesis 1:28). By "be fruitful", God meant much more than "have lots of children!" Like a farmer, we should seek fertile ground to plant seeds, where "seed" is an analogy to His word. If you think about it, the Great Commission (Matthew 28: 18-20) is basically a restatement of Genesis 1:28. It is a "sending", giving instructions to Christians on how God wants us to live and bear fruit for His glory.

Notice also how both Genesis 1:28 and Matthew 28:18-20 tell us to "go", but then there is little instruction after this! Genesis says "rule over", or "have dominion". Matthew says "Go", baptize, and teach others to observe all things Jesus commanded. These verses tell us *what* to do, just now *how* to do it. As Christians, we apply rules (deduction) that force us to find rules (induction)!

We also remember that in Genesis 1:26, God proclaims He created humans, and only humans, in His image. Of all the creatures He made, to Him we are a "kind of firstfruits." (James 1:18). Think about that. You were created in the image of God, whose creativity is boundless! And since you are "in His image", that means you have some amazing creative abilities, too! You just need to pray and work hard to develop them. God designed you to be creative, to discover, and to rule wisely!

Construction: You learned in Lesson 9 how Euclid used deductive reasoning to develop 465 theorems, or propositions, all from just 5 axioms, 5 postulates, and 23 definitions. Now, think about Euclid, sitting at a desk over 2,000 years ago, with pen and papyrus, compass and straightedge. Do you think the rules and definitions just popped into his head? Most certainly not! First, he had to do a lot of writing, drawing, reading and sketching before discovering them. In other words, he had to think inductively first. Then, he could apply these rules to discover new truths (deductive reasoning).

Reading the *Elements*, one thing we know Euclid figured out was how to construct a perfect equilateral triangle. Why don't you pretend like you are Euclid, and use just your compass and straightedge to draw an equilateral triangle. Refer to Lesson 9 if you forgot what an equilateral triangle is. Spend about 5 minutes on this. Here is a hint: start by using a straightedge to draw a line segment in the middle of your paper. Don't look ahead to 10B yet though!

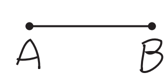
Optional: Use Geometer's Sketchpad to construct an equilateral triangle.

10B Deductive Reasoning and Proofs

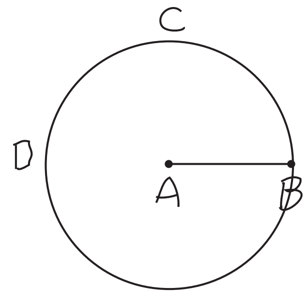
How did you do? Were you able to find a rule for drawing equilateral triangles? Actually, this is Proposition 1 in Euclid's *Elements*! Here's how he did it by applying definitions, axioms, and postulates.

Proposition 1: Given a line segment, construct an equilateral triangle.

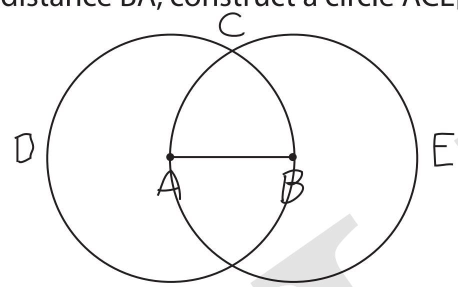
1) Construct line segment AB[Given].



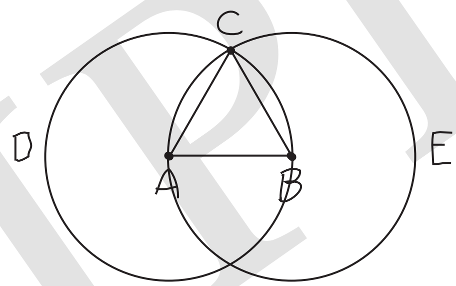
2) With center A and distance AB, construct a circle BCD[Post. 3].



3) With center B and distance BA, construct a circle ACE[Post. 3].

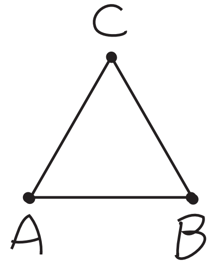


4) From point C, where the circles intersect, construct line segments CA and CB[Post. 1].



5) Now, since point A is the center of circle CBD, AC = AB [Def. of circle].
6) Also, since point B is the center of circle ACE, BC = BA [Def. of circle].
7) But CA was also proved equal to AB. Therefore, CA must also equal CB, since things equal to the same thing are equal to each other [Axiom 1].
8) Therefore, the three line segments CA, AB, and BC are equal to each other, and triangle ABC is therefore an equilateral triangle, being what was required to do.

As a final step, you can erase the circles to show just the equilateral triangle:



The 8 steps shown above are often described as a *mathematical proof*, a deductive argument where rules and definitions are applied to reach a logical conclusion. For example, the 8 steps shown above would not be nearly so convincing if the definitions, postulates and axioms inside each set of brackets were not included. **A proof requires a reason for every step taken**.

Now, think about what you just did. You used a compass and a straightedge to create an equilateral triangle whose sides were exactly equal to each other! Well, if you drew perfectly, the sides would all be exactly equal. Of course, noboby's perfect, but we can certainly get close to perfection, and performing constructions helps us realize that. By learning to apply rules to acheive perfection in little things like triangles, you will learn how to reach for perfection in bigger things, like playing a sport, running a business, raising a family, and most importantly, loving and serving Christ.

One of the amazing things about mathematics is just how close to perfection we can get just by following a few simple rules. Building on a foundation of self-evident truths, postulates, and definitions, we can discover new truths! Now, you will probably never need to know how to construct an equilateral triangle for your job, for a college entrance exam, etc. However, your life will be filled with opportunites where you need to solve problems by following specific directions (deductive reasoning). At other times there may be no specific directions, but you will solve the problem, creating some useful rules along the way (inductive reasoning).

Besides rules, definitions, and tools, another important component is skill. How did your equilateral triangle construction turn out? A common saying is "practice makes perfect", so just like a basketball player improves over time by shooting lots of baskets, you can improve your construction skills through practice. Try constructing an equilateral triangle using Proposition 1 again.

Practice Set 10 (subscripts tell you which lesson each problem came from)

You may use a calculator for problem 18 only.

1_{10}. Rearrange the steps shown below in the correct order to complete the proof of Euclid's Proposition 1. Don't rewrite everything, just write the letters A,B,C, etc. in the correct order.

A)With center A and distance AB, construct a circle BCD[Post. 3].
B) With center B and distance BA, construct a circle ACE[Post. 3].
C) Construct line segment AB[Given].
D) Now, since point A is the center of circle CBD, AC = AB [Def. of circle].
E) Also, since point B is the center of circle ACE, BC = BA [Def. of circle].
F) Therefore, the three line segments CA, AB, and BC are equal to each other, and triangle ABC is therefore an equilateral triangle, being what was required to do.
G) From point C, where the circles intersect, construct line segments CA and CB[Post. 1].
H) But CA was also proved equal to AB. Therefore, CA must also equal CB, since things equal to the same thing are equal to each other [Axiom 1].

2_{10}. Using a compass and straightedge, construct an equilateral triangle. Use Euclid's Proposition 1 as a guide.

3_{10}. Which of the following Scriptures commands us to use our inductive reasoning abilities?

a) John 3:16 b) Exodus 20:2-17
c) Genesis 1:26-28 d) Ephesians 2:8-9

4_{10}. Which of the following is about applying rules? A) Deductive reasoning B) Inductive reasoning.

5_{10}. Theorems are also called a) axioms b) postulates c) propositions d) proofs

6_{9}. Match Euclid's axioms. Write A1, A2, A3, A4, or A5 in the blank:

a) The whole is greater than the part:______
b) If equals be added to equals, the wholes are equal:_____
c) If a = c and b = c, then a = b:____
d) Things which coincide with one another are equal:_____
e) If equals be subtracted from equals, the remainders are equal:_____

7_{9}. Match Euclid's postulates. Write P1, P2, P3, P4, or P5 next to each drawing:

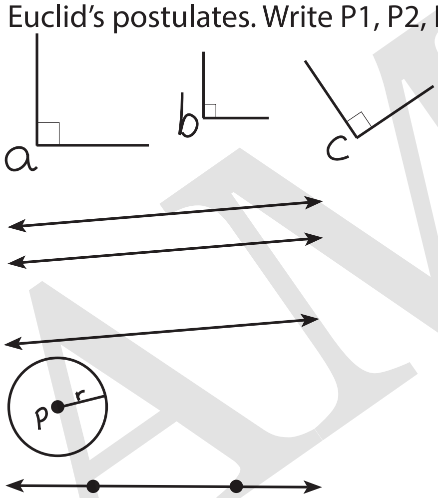
a)

b)

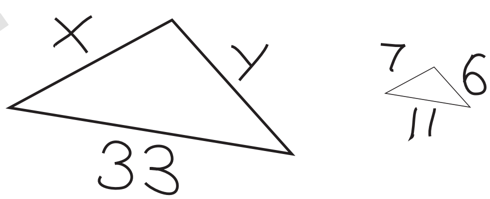
c)

d)

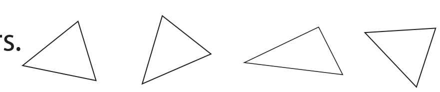
e)



8_{9}. Find x and y.



9_{9}. Circle the shape that is not congruent to the others.



10_{9}. True or False. A trapezoid is also a parallelogram.

11_{8}. If five less than three times a number is 19, what is the number?

12_8. Solve. 3x - 2 - 5x = 2x + 9

13_8. Expand $x^2(3x - y^2)$

14_8. Simplify by adding like terms.

$$6x^2y^0 + 5x^2 + \frac{3x^3y}{xy}$$

15_7. Evaluate st - t if s = 3 and t = 4

16_7. Simplify $\frac{3x^2y^3}{x^2y^2z}$

17_6. 1 knot = 1 nautical mile per hour. If the sailing ship traveled 60 nautical miles in 5 hours, what was its average speed in knots?

18_6. Fill in the empty cells of the following table: Round missing decimal and percent values to 1 decimal place. You may use a calculator.

Fraction	**Decimal**	**Percent**
1/10		
	$0.1\overline{6}$	
		24%

19_5. Divide. Round answer to 2 decimal places. 118 ÷ 3

20_3. Simplify: $-6^2 + (-3)^2 - |-5|(4-2)^2$

Lesson 25 Computer Mathematics

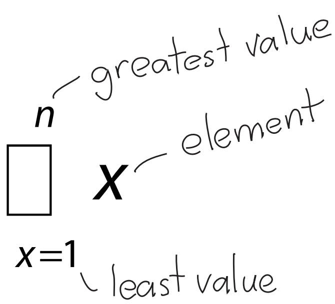
The intersection of continuity and discreteness is at the heart of what we will refer to as Computer Mathematics. Have you ever zoomed in on a digital photo on your computer, and noticed how the image becomes "pixelated?" That is because the computer programs inside the camera were designed to capture the image as a series of rows and columns of tiny squares called pixels. Each pixel is assigned a specific color (Photo by Dr. Shormann).

Rules and Definitions

Rules

- **Determinant:** The determinant of a 2 x 2 matrix of the form $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ equals ***ad-bc***.

- **Summation notation**: Note that the element is like a function, and can be any single-variable relationship (x^2, 1/t, $3b^5$+b, etc.).



Definitions

- **bit:** A digit in a binary number system. It can have two values, 1, or 0. In computer memory, a bit is a small electrical switch which is either on (value 1) or off (value 0).
- **byte:** In computer memory, a byte equals 8 bits.
- **pixel:** Abbreviation for "picture element," the small, discrete elements of digital

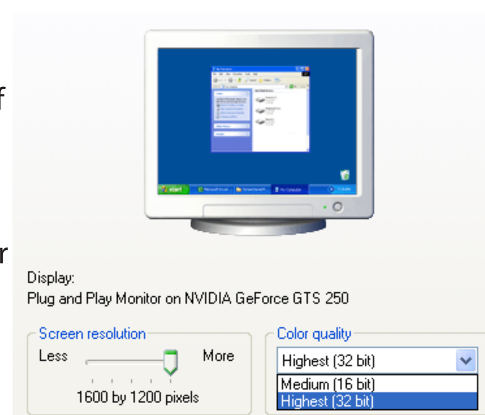
photography and computer/television screens containing color information.
- **matrix:** A two-dimensional array consisting of rows and columns of numbers.
- **array:** Like a matrix, but not limited to two dimensions. Used in computer data storage.
- **sequence:** Numbers ordered in a way that they form a definite pattern.
- **series:** The indicated sum of a sequence.

25A Pixels, Bits, and the Binary Numeral System

As we come to our last introductory lesson, I hope you are seeing how God's attributes are clearly displayed in mathematics, and I hope you are responding with a greater love for Him and desire to understand more about His word and works! His fingerprints are all over mathematics, especially regarding unity and diversity. The discovery of calculus helped humans think in new ways about how the reality of continuity and discreteness helps us understand nature, and be creative ourselves.

Understanding continuity and discreteness helps us understand the design and function of computers and components related to them, such as computer monitors. We can also understand some computer basics by studying digital photos. Photographs represent an image of an instant captured from the continuous flow of time. When a digital camera takes a picture, the image is stored in an array of pixels, which can then be displayed in two dimensions on a computer screen. In the photos above, you can see how the zoomed-in digital photo of the bear becomes "pixelated", revealing the individual color squares, or pixels. Each pixel displays color information.

You can probably imagine that the smaller the pixels, the more continuous an image will look. Larger pixels make an image look more "grainy" and unrealistic, while smaller pixels make it look more realistic. This is actually the difference between newer "HD" computer and television screens and older "SD" screens. HD screens have more pixels packed into the same amount of screen, resulting in better resolution. The better the resolution, the less grainy, and more continuous an image appears.



But what is a pixel anyways? Well, pixel is short for "picture element." One purpose of a computer's graphics card is to store and process information related to each individual pixel. In the figure to the right, you can see that typical display settings for a computer monitor allow adjustments of both resolution (number of pixels) and color quality (number of possible colors).

The simplest way to think about what computers do is that they store and process discrete packages of information. The information is stored electronically, so the more information a computer must process each second, the more electrical power is required. Computers are also limited by how efficiently they can process information. If a computer is running slowly, you can improve its speed by reducing the resolution, and the color quality, which will reduce the amount of information it needs to process. Another option is to purchase a computer with more memory and a better graphics card, which improves processsing efficiency.

In the display information shown on the preceding page, the "1600 x 1200" means 1,200 lines containing 1,600 pixels each, or 1,920,000 pixels! That's a lot of pixels, but not as many as some modern television screens.

Color quality is a little more difficult to understand, and involves the use of the binary number system you learned about in Lesson 2. First, consider the fact that each pixel is a combination of three primary colors, red, green and blue. Computers store information in bytes, which is 8 bits of information, or 8 digits in the binary system.

bits	8	7	6	5	4	3	2	1
base 10 equivalent	$2^7=128$	$2^6=64$	$2^5=32$	$2^4=16$	$2^3=8$	$2^2=4$	$2^1=2$	$2^0=1$

If you add all the values above, you get a maximum of 128+64+32+16+8+4+2+1 = 255. Think of these as 255 shades of color that you have assigned a number to. Add 0 as a possible number and you get 256 colors per byte. Now, since each pixel is a combination of red, green and blue, the computer can display at most 256 x 256 x 256 = 16,777,216 different colors! 256 also equals 2^8, so the color quality, or "depth" is referred to as $2^8 \times 2^8 \times 2^8 = 2^{24}$, or "24-bit" for short. Combine another 256, or 2^8, options for transparency, and you get the 2^{32} = "32 bit" color quality. On the figure shown, this is referred to as the highest quality setting, because it takes full advantage of the computer's graphics capabilites.

The "medium" quality setting reduces the amount of color options, and therefore the amount of information needing processing. Lower color quality settings are helpful sometimes when working with video, as they require a smaller amount of information processing each second.

Example 25.1 How much memory, in megabytes (MB) is required to display a medium quality image using 1600 x 1200 resolution?

solution: To understand how much memory is required to display an image, think back to the fact that 8 bits equals 1 byte of memory. Since 8 bits equals 1 byte, 16 bits equals 2 bytes, so each pixel of a medium quality image must process 2 bytes of information. The total memory is therefore

2 x 1600 x 1200 = 3,840,000 bytes = 3.84 Megabytes = **3.84 MB.**

Example 25.2 How much memory, in megabytes, is required to display a high quality image using1600 x 1200 resolution?

solution: 32 bits would equal 4 bytes, so the amount of memory doubles from Example 25.1 to
4 x 1600 x 1200 = 7,680,000 bytes = **7.68 MB.**

Now, connect the idea of reducing pixel size to the idea of infinitesimals in calculus. Does it seem strange to you that, the smaller the value of either, the closer the result matches reality? It just amazes me how we can get closer to understanding and modeling reality, by breaking it down into a series of instantaneous events. It shows us how even the most discrete of events are important to God! In fact, Scripture even refers to the importance of something that seems so insignificant to us as an "instant in time," but for God, that is all it takes to transform us from corruptible to incorruptible, mortal to immortal (I Corinthians 15: 52-53). Remember too what Leonard Euler said, that every instant, God is putting us in situations to bring about our salvation. In an instant, God can transform us from doubting Him to trusting Him. In an instant, He can transform us from a life bent on destruction and Hell, to a life of eternal joy. And we don't have to fully comprehend what "an instant" means. We just need not reject Him when He reaches out His hand to lead us across the divide from doubt to faith, via Christ our mediator (I Timothy 2:5).

At some point, every rational person must recognize the faith required to do mathematics. This is certainly not a saving faith by any stretch, but more of an acknowledgement that nature operates in ways outside of our abilities to measure it. Certainly, the believer and unbeliever alike can "have faith" in things like infinitesimals, and from that, understand much about creation. However, an unbeliever in this situation is like the hypocrite Jesus describes in Luke 12:56, who has a good understanding of how the natural world operates, but is willfully ignorant of His actions throughout history (2 Peter 3:5).

Scripture makes it clear that God's attributes have been clearly visible by humans since creation's beginning (Romans 1:20). And one thing I hope is clear to you is that humans simply cannot rely on our intellect alone to make sense of reality. We simply have to trust Him to be in charge of the Universe and then get out there and explore it, and by so doing glorify and enjoy Him!

25B Sequences and Series

<u>Sequences</u>

We know that computers require discrete pieces of information, so it should make sense that computers make use of specific patterns of numbers called sequences. The two most common types of sequences are *arithmetic* sequences and *geometric* sequences. In an arithmetic sequence, each term is separated by a common difference. In a geometric sequence, each term is separated from the others by a common ratio.

Example 25.3 Find the common difference for the following arithmetic sequences:

a) 2, 4, 6, 8, 10, ... b) 6, 3, 0, -3, -6, -9, ...

solution: The common difference will be the same between each term and the one before it.

a) The common difference is **2** b) the common difference is **-3**

Example 25.4 Find the common ratio for the following geometric sequences:

a) 2, 4, 8, 16, 32, ... b) 6, 3, 3/2, 3/4, 3/8, 3/16, ...

solution: The common ratio will be the same between each term and the one before it.

a) The common ratio is **2** b) the common ratio is **1/2**

Series

Series are the indicated sums of a sequence. Here we will consider only arithmetic and geometric series.

Example 25.5 Find the sum of the first 5 terms of an arithmetic series that begins with 5 and has a common difference of 3.

solution: An arithmetic series is like a sequence with plus signs between the numbers.

5 + 8 + 11 + 14 + 17 = **55**

Example 25.6 Find the sum of the first 5 terms of an geometric series that begins with 2 and has a common ratio of 2.

solution: A geometric series is like a sequence with plus signs between the numbers.

2 + 4 + 8 + 16 + 32 = **62**

25C Sums

Sums are a lot like series, except that they don't necessarily follow an arithmetic or geometric pattern. Let's do some examples using an arithmetic pattern, a geometric pattern, and a different pattern.

Example 25.7 Find $\sum_{x=1}^{4} 2x$

solution: In a summation, integer values are substituted in for x, and the results are summed together, like this:

2(1) + 2(2) + 2(3) + 2(4) = 2 + 4 + 6 + 8 = **20**

Example 25.8 Find $\sum_{x=0}^{7} 2^x$

solution: As you substitute values in to complete the summation, notice the similarity to this and the example of an 8-bit system in 25A. Notice also that it forms a geometric series with a common ratio of 2:

$2^0 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6 + 2^7$ = 1 + 2 + 4 + 8 + 16 + 32 + 64 + 128 = **255**

Example 25.9 Find $\sum_{t=1}^{3} \frac{2^t}{t}$

solution: This one is a bit more complicated, but just substitute and add as before:

$$\frac{2^1}{1}+\frac{2^2}{2}+\frac{2^3}{3}=$$

$$\frac{2}{1}+\frac{4}{2}+\frac{8}{3}=$$

$$2+2+2\frac{2}{3}=6\frac{2}{3}$$

Summation notation was developed by Leonhard Euler, who chose to use the capitalized Greek letter sigma (∑) to represent summation. Symbolic notation saves us from having to explain in writing which steps to repeat each time. Just by recognizing the notation, we know our first step is to substitute integer values into the given element over the range shown, and then sum each individual solution together.

I hope you noticed how both arithmetic and geometric series can be represented using summation notation and a simple formula. Perhaps you also considered the fact that, for the above sums, if you were asked to sum out to a very large final value of x or t, that it would become extremely tedious and difficult to keep track of. But that is where computer programming comes in handy! A computer program could be written to mimic the steps of a summation and keep track of it.

25D Matrices

Computer programs retrieve and store data contained in matrices. Because computers use matrices, it is important to understand some basic mathematical relationships regarding matrices. For now, we will not work with anything larger than a 2 x 2 matrix (2 rows by 2 columns). Just know that it is possible to work with much, much larger matrices than 2 x 2!

Example 25.10 Find the determinants of the following matrices.

a) $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$ b) $\begin{vmatrix} 0 & 2 \\ 4 & 1 \end{vmatrix}$ c) $\begin{vmatrix} 2 & 1 \\ -3 & 5 \end{vmatrix}$

solution: Recall from the Rules section that the determinant of a 2 x 2 matrix of form $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ equals ***ad-bc***. We will refer to each position as a *cell*.

a) $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$ = 1(4) - 2(3) = 4 - 6 = **-2** b) $\begin{vmatrix} 0 & 2 \\ 4 & 1 \end{vmatrix}$ = 0(1) - 2(4) = 0-8 = **-8**

c) $\begin{vmatrix} 2 & 1 \\ -3 & 5 \end{vmatrix}$ = 2(5) - 1(-3) = 10 + 3 = **13**

Example 25.11 Add or subtract the following matrices.

a) $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} + \begin{vmatrix} 0 & 2 \\ 4 & 1 \end{vmatrix}$ b) $\begin{vmatrix} 2 & 1 \\ -3 & 5 \end{vmatrix} - \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$

solution: Addition and subraction is simply a matter of finding the sum or difference between corresponding cells.

a) $$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} + \begin{vmatrix} 0 & 2 \\ 4 & 1 \end{vmatrix} = \begin{vmatrix} 1+0 & 2+2 \\ 3+4 & 4+1 \end{vmatrix} = \begin{vmatrix} 1 & 4 \\ 7 & 5 \end{vmatrix}$$

b) $$\begin{vmatrix} 2 & 1 \\ -3 & 5 \end{vmatrix} - \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} 2-1 & 1-2 \\ -3-3 & 5-4 \end{vmatrix} = \begin{vmatrix} 1 & -1 \\ -6 & 1 \end{vmatrix}$$

Practice Set 25 (subscripts tell you which lesson each problem came from)

1_{25}. How much memory, in kilobytes (KB), is required to display a medium quality image using 640 x 480 screen resolution?

2_{25}. Find the next 3 terms for the following arithmetic sequence. What is the common difference?

12, 16, 20, 24, ____, ____, ____

3_{25}. Find the next three terms for the following geometric sequence. What is the common ratio?

3, 6, 12, 24, ____, ____, ____

4_{25}. Find the sum of the first 5 terms of an arithmetic series that begins with 1 and has a common difference of 4.

5_{25}. Find the sum of the first 3 terms of a geometric series that begins with 7 and has a common ratio of 3.

6_{25}. Find $\sum_{x=1}^{4} 3x$

7_{25}. Find $\sum_{x=0}^{3} 3^x$

8_{25}. Find $\sum_{n=2}^{4} 2n+4$

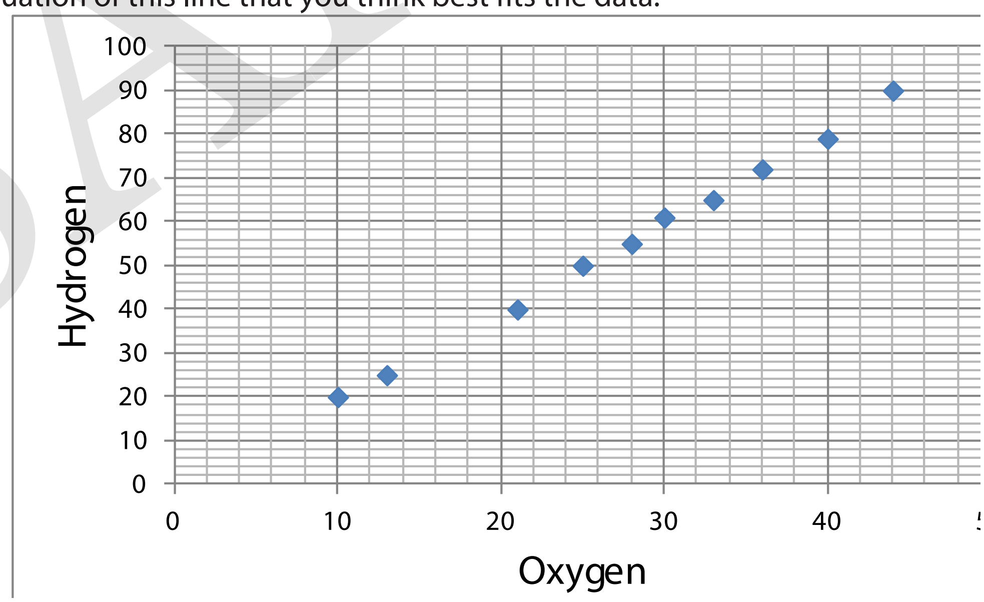
9_{25}. Find the determinant of $\begin{vmatrix} 1 & 1 \\ 3 & 3 \end{vmatrix}$

10_{25}. Find the sum of $\begin{vmatrix} 1 & 1 \\ 3 & 3 \end{vmatrix} + \begin{vmatrix} 4 & 4 \\ 1 & 1 \end{vmatrix}$

11_{24}. What is the probability of rolling a 6-sided die and having it land on a 1 or a 6?

12_{24}. What is the probability of rolling a 6-sided die two times, and having it land on 3 both times?

13_{24}. Use a pencil and straight edge to draw a line through the data plotted below that you think best fits it. If you are using the digital version of the book, just put your paper on your screen and trace the graph, then draw your line. Afterwards, write the equation of this line that you think best fits the data.



Use the following data for problems 14-15. The data is a set of masses, in grams, of individual eggs from a carton of 1 dozen eggs.

72, 67, 74, 68, 75, 72, 74, 70, 70, 69, 68, 73

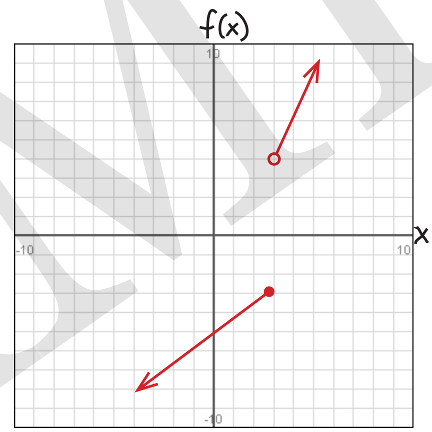
14_{23}. Organize the data into a table that sorts the data by 2-gram increments, as shown.

Group	Values
67-68.9	
69-70.9	
71-72.9	
73-74.9	
75-76.9	

15_{23}. Use the table created in Problem 14 to make a histogram. Does the data follow a normal distribution?

16_{22}. If $f(x) = 3x^2$, find $f'(3)$. Hint, evaluate $f'(x)$ then $x = 3$.

17_{20}. For the graph shown, find $\lim_{x \to 3^+} f(x)$



18_{15}. Which of the following functions is continuous?

a) $f(x) = x^2 - 3$ b) $g(x) = \frac{1}{x^2 \square 3}$ c) $k(x) = \frac{1}{x^2}$

19_6. Find the missing number. 5 : 2 :: 25 : ?

20_2. Convert the binary number 111000 to base 10.

Lesson 50 Expanding Squared Binomials, Euler Word Problems

Review: Lessons 8, 37, 38 on expanding; Lessons 46 and 47 on solving algebraic equations.

50A Expanding Squared Binomials

In Lesson 7, you learned that a *binomial* is just a polynomial with two terms. You already know how to expand all kinds of polynomials. We just haven't covered squared binomials yet. These show up in many places, especially when working with parabolas (Lesson 16). And parabolas are important for you to know about because, among other things, a lot of natural phenomena follow parabolic patterns.

Example 50.1 Expand. $(x + 2)^2$

solution: The exponent of 2 is the simplified way of writing $(x + 2)$ as a factor twice. So, write the binomial twice, then expand as usual.

$(x + 2)(x + 2) =$
$x(x + 2) + 2(x + 2) =$
$x^2 + 2x + 2x + 4 = \mathbf{x^2 + 4x + 4}$

Example 50.2 Expand. $(a - 5)^2$

solution: In the middle step below, instead of writing it out, try doing this part mentally, and just write the third step and add like terms.

$(a - 5)(a - 5) =$
$a(a - 5) - 5(a - 5) =$
$a^2 - 5a - 5a + 25 = \mathbf{a^2 - 10a + 25}$

Example 50.3 Expand. $(x + y)^2$

solution: Again, try to do the middle step mentally.

$(x + y)(x + y) =$
$x(x + y) + y(x + y) =$
$x^2 + xy + xy + y^2 = \mathbf{x^2 + 2xy + y^2}$

50B Euler Word Problems

These are word problems straight from Leonard Euler's book, *Elements of Algebra*. They will require you to create and solve equations like those covered in Lessons 46 and 47. One of the "rights of passage" of algebra is for you to be able to create algebraic equations or functions from information given to you in sentence or paragraph form. Euler's textbook contains a few dozen word problems that he thought would help his students understand and apply basic algebra principles. So, be confident that, if you can learn to solve problems written by the greatest mathematician ever, then you must have some pretty awesome math skills, too!

NOTE: All the Euler word problems covered in this lesson can be solved by setting up one equation with one unknown.

Example 50.4 A father leaves 1600 pounds of gold to be divided among his three sons in the following manner: The eldest is to have 200 pounds more than the second, and the second 100 pounds more than the youngest. How much does each receive?

solution: If we use x to describe the youngest son's share of gold, then we have
Youngest = x
Second = $100 + x$
Eldest = $200 + 100 + x = 300 + x$

Now, just add them together and set them equal to 1600, and solve for x:

$x + 100 + x + 300 + x = 1600$
$3x + 400 = 1600$
$3x = 1200$
$x = 400$

Be careful to answer the question correctly. The question wasn't "solve for x," the question was "How much does each receive?"

Youngest = 400 lbs
Second = 500 lbs
Eldest = 700 lbs

Example 50.5 A father leaves four daughters, who share his property in the following manner: the eldest takes half the property, minus 3,000 acres; the second takes one-third, minus 1,000 acres; the third takes exactly one-fourth of the property; and the youngest takes 600 acres and one-fifth of the property. How much property did the father own?

solution: If we use x to describe the total amount of property, then

Eldest = $\frac{1}{2}x - 3000$ Second = $\frac{1}{3}x - 1000$

Third = $\frac{1}{4}x$ Youngest = $\frac{1}{5}x + 600$

The sum of the four pieces of property has to equal x, so

$$\frac{1}{2}x - 3000 + \frac{1}{3}x - 1000 + \frac{1}{4}x + \frac{1}{5}x + 600 = x$$

Now, get rid of the fractions and solve for x:

$$-3400 + \frac{1}{2}x + \frac{1}{3}x + \frac{1}{4}x + \frac{1}{5}x = x$$

$$60\left(-3400 + \frac{1}{2}x + \frac{1}{3}x + \frac{1}{4}x + \frac{1}{5}x = x\right)$$

$$-204{,}000 + 30x + 20x + 15x + 12x = 60x$$

$$77x = 60x + 204{,}000$$

$$17x = 204{,}000$$

$$\boxed{x = 12{,}000 \text{ acres}}$$

Practice Set 50 Use your best judgement as to when you should and shouldn't use a calculator. Use 3.14 for π.

1_{50}. Expand. $(x + 7)^2$

2_{50}. Expand. $(f - 1)^2$

3_{50}. Expand. $(x - y)^2$

4_{50}. A father leaves his four sons 8,600 hectares of land, and, according to the will, the share of the eldest is to be double that of the second, minus 100; the share of the second is three times as much as the third, minus 200, and the third is to receive four times as much as the youngest, minus 300. How many hectares does each son receive?
Hint: set x= youngest's amount of property.

5_{49}. Find the equation of the line that passes through the point (-1,-3) and is parallel to $y = -2x + 2$.

6_{49}. Find the equation of the line that passes through the point (-2,5) and is perpendicular to $3x + 6y = 7$.

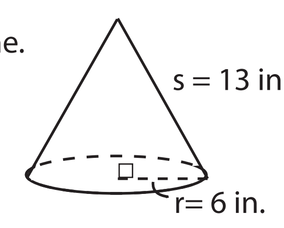
7_{48}. Find the equation of the line that passes through the points (1,1) and (5,6).

8_{47}. Solve for *a*. $\frac{6}{a} + b = c$

9_{46}. Solve for h. $3h - 2k = 2h - 17$

10_{44}. Use 1 unit multiplier to convert 30 miles to kilometers.
Round answer to 1 decimal place.

11_{41}. Find the lateral surface area of this cone.
Round answer to 2 decimal places.



12_{39}. Simplify. $\dfrac{t^3(x+\Delta x)^2}{x+\Delta x}$

13_{33}. Multiply. Write answer in scientific notation. $(2 \times 10^7)(4 \times 10^{-1})$

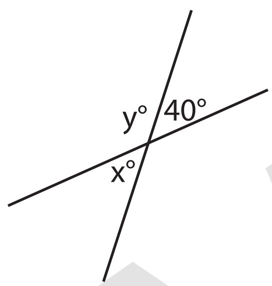
14_{28}. The ratio of force to area in the hydraulic system was 700 to 3. Find the force when the area was 9.

$15_{49,8}$. (CLEP College Math) The sum of five times a number *n* and the negative reciprocal of *n* is equal to 11. Which of the following equations represents this sentence algebraically?

A) $5\left(n-\dfrac{1}{n}\right)=11$ B) $5n-\dfrac{1}{n}=11$ C) $5n+\dfrac{1}{n}=11$ D) $-\dfrac{5}{n}=11$

16_{15}. If $g(x) = 2x - x^2 - 7$, find g(x) when x = 2.

17_{12}. Find x and y.



18_{11}. Find the sum of the interior angles of a polygon with 7 sides. If necessary, review your rules in Lesson 11.

19_{8}. Factor. $3x^2y + 15x^2y^2 + 12x^3y^2$

20_{6}. Pressure equals force per unit area. If 60 lbs of force were exerted on a 3 square inch surface, what was the pressure in pounds per square inch?

Lesson 75 Quadratic Equations: Standard Form and Factoring

Review: Lesson 16, 51

Rule

The standard form of a quadratic equation is

$$ax^2 + bx + c = 0$$

a, *b*, and *c* are coefficients (numbers), and *x* is the variable.

Definition

quadratic equation: An equation containing a second degree polynomial of the form $ax^2 + bx + c$.

75A Standard Form of Quadratic Equations

We've mentioned many times already how Leonhard Euler's 1765 textbook, *Elements of Algebra*, serves as a pattern for modern algebra courses, including this one. Euler dedicated a lot of his text to equations of the first, second, and third degree, where *degree* refers to the highest power a variable has in the equation. Each equation type also has a corresponding geometric name, as shown below:

Algebraic name	Geometric name
First degree equation	Linear equation
Second degree equation	Quadratic equation
Third degree equation	Cubic equation

For second degree equations, the geometric name, quadratic, comes from quadrilateral. Think about it, rectangular shapes are also called quadrilaterals, and to find their area you multiply length by width, or x·x, and get x^2 (x -squared). Likewise, finding the volume of a rectangular, or cubic solid requires multiplying three lengths (x-cubed). Do you have a nickname? Just think of these geometric names as nicknames for the algebraic equations described above.

You have already worked a lot with second degree equations and functions, especially in analytical geometry, where the graph of a second-degree function is a parabola. And the reason a standard quadratic equation is set equal to zero has to do with its graph. But don't worry about why that is right now, just get familiar with working with them.

Example 75.1 Write the following quadratic equations in standard form.

a) $3x^2 - 2x = -2$ b) $4x+1=-x^2$ c) $2x + 5x^2 = 0$

solution: Just rearrange them into their standard form, $ax^2 + bx + c = 0$. And if one or more coefficients equals zero, don't write them.
a) $3x^2 - 2x + 2 = 0$
b) $x^2 + 4x + 1 = 0$
c) $5x^2 + 2x = 0$

75B Factoring Quadratic Equations

Did you notice the standard form of a quadratic equation is also a trinomial? Since Lesson 51, you've had a lot of practice factoring trinomials into two binomials. This is a very important step in solving quadratic equations. We will focus on solving quadratic equations in Lessons 76, 90 and 91. For now though, practice recognizing second degree trinomials within equations, and factoring them.

Example 75.2 Factor the following.

a) $x^2 + 5x + 6 = 0$ b) $x^2 + 2x - 3 = 0$ c) $x^2 + 10x = -25$

d) $x^3 - 6x^2 + 8x = 0$

solution: All you need to do is factor the trinomial into two binomials:

a) $\mathbf{(x + 3)(x + 2) = 0}$
b) $\mathbf{(x + 3)(x - 1) = 0}$
c) rearrange first: $x^2 + 10x + 25 = 0$
$(x + 5)(x + 5) = \mathbf{(x + 5)^2 = 0}$
d) Notice this is a cubic equation. Factor an x out first: $x(x^2 - 6x + 8) = 0$
$\mathbf{x(x - 4)(x - 2) = 0}$

Practice Set 75 Use your best judgement as to when you should and shouldn't use a calculator.

1_{75}. Write the following quadratic equation in standard form. $6x^2 - 3 = 5x$

2_{75}. Factor the following. $x^2 + 6x - 7 = 0$

3_{75}. Factor the following. $x^3 - x^2 - 30x = 0$

4_{74}. Find three consecutive integers such that 4 times the second equals the sum of the first and the third.

5_{72}. (Astronomy) The velocity (V) of a galaxy is thought to be directly proportional to its proper distance (D). Galaxy 1 had a velocity of 72 and a proper distance of 1. Find the proper distance of Galaxy 2 it its velocity equaled 144. (Units were ignored for the sake of simplification.)

6_{71}. Solve for x: $\frac{x-1}{x}+\frac{4}{3x}=\frac{1}{4}$

7_{70}. Use substitution or elimination to solve the following system. $\begin{cases} 2x+3y=23 \\ 5x-2y=10 \end{cases}$

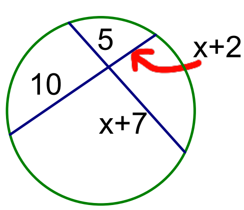
8_{69}. Estimate the future sum of a present value of $5,000 deposited into a savings account at 3% interest for 20 years, compounded continuously. Use 2.718 for *e*.

9_{67}. Rearrange the steps shown below in the correct order to complete the proof of Euclid's Proposition 2. Don't rewrite everything, just write the letters A,B,C, etc. in the order that correctly describes the proof. Then, using a compass and straightedge, construct Euclid's Proposition 2.

Proposition 2- To place at a given point a straight line segment equal to a given line segment.

Statements	***Reasons***
A) Construct equilateral triangle DAB	Proposition 1
B) Construct segment AB	Postulate 1
C) Construct segments AE and BF in line with DA and DB	Postulate 2
D) Construct circle CGH with center B and radius BC	Postulate 3
E) Construct circle GKL with center D and radius DG	Postulate 3
F) Point A, Line segment BC	Given
G) Since point B is the center of circle CGH, BC equals BG. Since point D is the center of circle GKL, DL equals DG. In these, DA equals DB. Therefore, the remainder AL equals the remainder BG	Axiom 3
H) But BC was also proved to equal BG; therefore, both AL and BC equal BG, and things equal to the same thing are equal to each other.	Axiom 1
I) Therefore, AL equals BC.	Q.E.D.

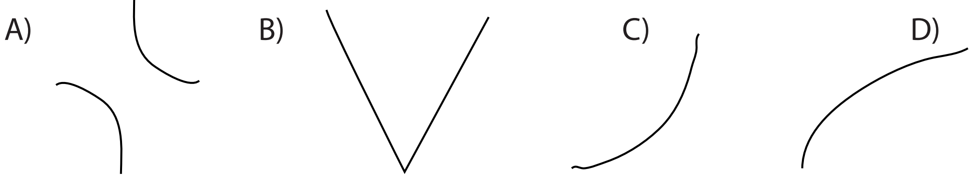
10_{66}. Solve for x.



11_{62}. Simplify. $\frac{\frac{1}{x}+3}{\frac{2}{t}}$

12_{59}. Convert (-3,3) to polar form.

13_{52}. Identify the function that corresponds with each shape.



14_{51}. Factor. $x^2 + 4x - 12$

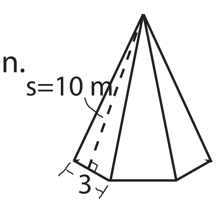
15_{60}. (SAT) The following table gives values of mass and volume for samples of continental crust and oceanic crust. Which type of crust is more dense?

Crustal type	Mass	Volume
Oceanic	13.2 g	4 mL
Continental	40.5 g	15 mL

A) Oceanic B) Continental C) Their densities are equal
D) Not enough information was given to determine which type of crust is more dense.

16_{48}. Write the answer to Practice Set problem 74.16 using functional notation. The problem is repeated below, but the choices are different.
For the tip of a robot arm to move at 1 meter per second requires a rotation rate of 2 rpm. A rotation rate of 4 rpm produces a tip speed of 2.1 mps. Which of the following models best describes the robot arm's tip speed (s) as a linear function of rotation rate (r)?
A) s(r) = 0.55r - 0.1 B) s(r) = 1.82r + 0.2 C) s(r) = 1.82r - 1.1 D) s(r) = 0.55r + 1.1

17_{41}. Find the lateral surface area of a prism whose base is a regular hexagon.



18_{28}. (Astronomy) The moon model was a 1/1,000,000 scale replica. If the model was 11.4 feet in diameter, what is the moon's actual diameter?

19_{20}. Evaluate. $\lim_{x \to 0^+} \frac{1}{x^2}$

20_{1}. Match the definition of mathematics with its author. Choices include: Euler, Kline, Sawyer, Shormann, Whitehead

A) The foundation of exact thought as applied to natural phenomena = _____________.
B) The classification and study of all possible patterns = ___________________________.
C) A God-given tool for measuring and classifying patterns and shapes = ___________.
D) A study of space and quantity = ____________________.
E) The science which investigates the means of measuring quantity = _____________.

Lesson 100 Chi-Square Test

Review: Lessons 23, 24, 77, 94

Rule

Chi-square Distribution Table					
Probability	Degrees of Freedom (df)				
p=	1	2	3	4	5
0.05	3.84	5.99	7.82	9.49	11.1

Definitions

Chi-square test: a statistical method assessing the goodness of fit between observed values and those expected values based on mathematical models.

null hypothesis - in a statistical test, the initial guess that there is no difference between observed values and those expected values based on mathematical models. In a chi-square test, we accept the null hypothesis if $\sum$ is less than or equal to the appropriate value found in the Chi-Square Distribution Table.

degrees of freedom (df) - a value that refers to the number of independent groups of observations in a sample (n), minus the number parameters under consideration. For our purposes, df = n - 1.

Congratulations, you are on the last lesson of Shormann Mathematics, Algebra 1! I hope your love for learning has increased, and not decreased as a result of this course. And while I am sure you are glad that you are almost finished, I hope you are thinking ahead to next year as well, and what math and science you will be learning.

In some science courses, biology in particular, your laboratory activities may involve using the Chi-square test (χ^2 test). Think back to Lesson 77 and Punnett Squares, which you used to predict the possible outcomes of genetic crosses. A common high school (and college) biology lab experiment involves crossing different genetic strains of fruit flies (*Drosophila spp.*), using Punnett Squares to predict results, and then comparing those to results collected from an actual experiment. We call the Punnett Square values the *expected results*, and the experimental data the *observed results*.

What does this have to do with the Chi-square test? Well let's say, for example, you did a fruit fly experiment, and of the 40 offspring grown in your experiment, you *expected* 75% of the flies to have normal eyes, and 25% to have sepia-colored eyes. You *observed* that 80% had normal eyes, and 20% had sepia eyes. So, does that mean the Punnett square is completely wrong, and a poor predictor of genetic crosses? Is that difference significant? You can't just go tell somebody, "there's no significant difference because I say there's no difference!"

Just like with mathematical proofs, you have to give a reason why you say there is or isn't a difference between expected and observed results. And that's what statistical tests like the Chi-square test do for us. They give us a reason to support what we mean by a *significant* difference. In the Chi-square test, we would say the observed and expected values in a test are the same if the probability of observed and expected values being *different* due to chance alone is 5% ($p = 0.05$) or more. In other words, the reason for the differences we see are from chance and not some other factor. Likewise, the probability of the samples being *the same* due to chance alone is less than 5%, which means the model (expected results) does a good job of predicting actual (observed) results. We might also say that we are > 95% confident the model predicts actual data. Let's do a Chi-square test on our fruit fly example, plus another.

Example 100.1 A genetics experiment involving simple dominance in fruit flies produced the observed and expected values shown in the table. Complete all the calculations in the table, and determine whether or not to accept the null hypothesis (no difference in observed and expected phenotypes).

Degrees of Freedom = **A**					
Phenotype	#observed	#expected	difference (o-e)	$(o\text{-}e)^2$	$(o\text{-}e)^2/e$
Normal	32	30	B	D	F
Sepia	8	10	C	E	G
				Σ=	H
				Accept or reject null hypothesis?	I

solution: With linear regression, you were introduced to the method in Lesson 94, without going into great detail about all the statistics functions used to create the data. We will do the same here. Notice letters A-I in the table above. Each letter represents a calculation you need to perform. We will do them in order now.

A) Your experiment includes 2 phenotypes, normal and sepia, so $n = 2$, and $df = 2-1 = 1$. Look at the chi-square distribution table, and make a note of the chi-square value for 1 degree of freedom (3.84). When you get to steps H and I, you will accept the null hypothesis if $H \leq 3.84$.

B) For normal eyes, o - e = 32 - 30 = 2
C) For sepia eyes, o - e = 8 - 10 = -2
D) For normal eyes, $(o - e)^2 = (2)^2 = 4$
E) For sepia eyes, $(o - e)^2 = (-2)^2 = 4$
F) For normal eyes, $(o - e)^2/e = 4/30 = 0.133$
G) For sepia eyes, $(o - e)^2/e = 4/10 = 0.4$
H) ∑ really means "sum" here, so just add F and G: 0.133 + 0.4 = 0.533
I) Since 0.533 < 3.84, we accept the null hypothesis. What that means is the differences between observed and expected results are small enough that chance alone accounts for them. Another way to think about it is that we can be more than 95% confident there's no statistically significant difference between observed and expected results, which means the Punnett Square (expected results) did a good job of modeling real data (observed results).

Here is the chart filled with the actual values

Degrees of Freedom = 1					
Phenotype	#observed	#expected	difference (o-e)	$(o-e)^2$	$(o-e)^2/e$
Normal	32	30	2	4	.133
Sepia	8	10	-2	4	.4
				∑=	.533
				Accept or reject null hypothesis?	accept

Example 100.2 The machine was designed to produce 3 colors of candies in the proportions shown below. As part of normal quality control measures, a sample of 100 candies was taken from a batch, which had the observed amounts shown. Complete the table, and determine whether the sample was significantly different from the expected results.

Degrees of Freedom = A					
Candy color	#observed	#expected	difference (o-e)	$(o-e)^2$	$(o-e)^2/e$
Red	29	20	B	E	H
Blue	40	50	C	F	I
Yellow	33	30	D	G	J
				∑=	K
				Accept or reject null hypothesis?	L

solution: The Chi-square test can be applied to any time you need to compare observed data from 2 or more independent groups with results expected from a mathematical model. The mathematical model could be a Punnett Square, or it may be a more complex algorithm used to program a candy-making machine. Either way, you are still comparing observed and expected values. The steps are the same as in Example 100.1, except that this example has results from 3 groups instead of 2.

A) df = 3 - 1 = 2. Look at the chi-square distribution table, and make a note of the chi-square value for 2 degrees of freedom (5.99). When you get to steps K and L, you will accept the null hypothesis if $H \leq 5.99$.
B) For red candy, o - e = 29 - 20 = 9
C) For blue candy, o - e = 40 - 50 = -10
D) For yellow candy, o - e = 33 - 30 = 3
E) For red candy, $(o - e)^2 = 81$
F) For blue candy, $(o - e)^2 = 100$
G) For yellow candy, $(o - e)^2 = 9$
H) For red candy, $(o - e)^2/e = 81/20 = 4.05$
I) For blue candy, $(o - e)^2/e = 100/50 = 2.0$
J) For yellow candy, $(o - e)^2/e = 9/30 = .3$
K) $\sum = 4.05 + 2.0 + 0.3 = 6.35$
L) Reject the null hypothesis, because 6.35 > 5.99

In Example 100.2, imagine what these results might mean in a real situation. There are a number of possible causes, but obviously, if the machine is designed to produce colored candies in specific proportions, and, statistically speaking, it's not, then there could be a malfunction in the machine. Maybe the mechanism is malfunctioning that sorts the candies before they are coated. Or perhaps an employee is stealing blue candies. Whatever the reason, the Chi-square test is a good mathematical tool for comparing actual results to expected ones.

Practice Set 100 Use your best judgement as to when you should and shouldn't use a calculator.

1_{100}. A genetics experiment involving simple dominance in fruit flies produced the observed and expected values shown in the table. The experiment is called a *dihybrid cross*, and it involved eye color and wing shape. Complete all the calculations in the table, and determine whether or not to accept the null hypothesis (no difference in observed and expected phenotypes).

Degrees of Freedom =					
Phenotype	#observed	#expected	difference (o-e)	$(o\text{-}e)^2$	$(o\text{-}e)^2/e$
normal wings and eyes	20	18			
normal wings, sepia eyes	3	6			
vestigial wings, normal eyes	5	6			
vestigial wings, sepia eyes	4	2			
				Σ=	
				Accept or reject null hypothesis?	

2_{99}. Which of the following gives all the values for which $|x-2| \not< 9$?
A) x ≥ 11, x ≤ -7 B) x ≥ 7, x ≤ -11 C) -11 ≤ x ≤ 7 D) -7 ≤ x ≤ 11

3_{98}. Construct a perpendicular line segment through a given point P. To begin, you can copy the example shown below, or draw something similar.

4_{97}. If a + 7 = b, find |a - b|

5_{96}. At 6:00 a.m., Captain Nemo left the island and headed east at 10 knots. At 9:00 a.m., Capt. Farragut left the same island and headed east. If Capt. Farragut was 20 nautical miles ahead at 2:00 p.m., what was his ship's speed? (1 knot = 1 nautical mile per hour).

6_{95}. Use the quadratic formula to find the complex roots of $x^2 - 6x + 10 = 0$.

7_{93}. Use upper rectangles with areas equal to $f(x) \cdot \Delta x$ to estimate the area under the curve of $f(x) = x^2$, on the interval [4, 6]. Partition the interval into 2 subintervals.

8_{92}. Use the quadratic formula to find the zeros of $3x^2 + 6x + 2 = 0$.

9_{90}. Find the sum of the first 8 terms of a geometric series whose 1st term is 1, and whose common ratio is $\frac{1}{2}$.

10_{82}. Olga had 19 nickels and dimes worth $1.50. How many of each type of coin did she have?

11_{74}. Find 3 consecutive even integers such that 2 times the sum of the first and third is 12 more than the second.

12_{66}. Given T = 20 °C, write a two-column proof that proves 20 °C = 68 °F. Use the formula, F = 1.8C + 32 (Lesson 18) in your proof.

13_{58}. Find the distance between (15, 16) and (18, 20).

14_{50}. Expand $(t - 10)^2$

15_{42}. Find the volume of a 5-m long pipe with an outside diameter of 4 m, and an inside diameter of 2 m. Use 3.14 for π. Round answer to 1 d.p.

16_{25}. Evaluate $\sum_{t=1}^{4} 5t$

17_{15}. Determine whether the set of ordered pairs represents a function or a relation. (1,1), (2,2), (2,3), (3,4), (4,5)

18_{10}. Applying verses like Genesis 1:28 and Matthew 28:18-20 remind Christians to apply their God-given _______________reasoning abilities to find rules.

19_{5}. 14% of 90 equals what number?

20_{1}. Mathematics is considered the language of __________________.